

# Charm production by cosmic muons

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## Abstract

Narrow muon bundles in underground detectors permit to study muoproduction reactions that take place in the surrounding rock. We analyze the relevance of a QED+QCD reaction, muoproduction of “open charm”. The contribution to double muon events is estimated to be 4 – 8% of the one due to QED “trident” process, for an ideal detector located under a rock depth of 3 km water equivalent, and an observation threshold of 1 GeV.

In recent years, there has been a certain experimental [1, 2] and theoretical interest [3, 4, 5, 6] on “narrow muon bundles” (multiple muons with a lateral separation less than a few meters) in underground detectors. These events have been observed as a peak at small lateral separation, and interpreted as an induced flux. In fact, the energetic muons that propagate underground in roughly  $\sim 1\%$  of the cases interact and produce other muons. Thence, an analysis of these events requires to consider high energy muoproduction processes, in the rock surrounding the detector.

Up to now, the process considered was the muon “trident” reaction [7]  $\mu Z \rightarrow \mu Z \mu^+ \mu^-$ , where a muon pair is formed in the field of the nucleus<sup>1</sup>. For muons propagating in high  $Z$  materials an amplification factor  $Z^2/A$  ( $= 5.5$  for standard rock,  $A=22$  and  $Z=11$ ) is present, due to coherent character of the reaction. This interpretation has been pursued since the first evidences obtained in cosmic ray experiments [9]. The trident reaction leads mostly to narrow bundles of three or two muons in an underground detector (one produced muon may stop before reaching the detector); a reference ratio in an ideal detector is of 3 double

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<sup>1</sup>It is assumed that an effective rejection of muoproduced  $\pi^\pm$ ,  $\gamma$ ,  $e^\pm$  ... can be achieved.

muons *per* triple muon, for a threshold of observation  $E_h = 1$  GeV, and a depth  $h = 3$  km w.e. of standard rock. Recent studies [5, 6], however, suggest that existing interpretations are insufficient to quantitatively account for the whole “narrow muon” data set.

In this work we analyze the role of another high energy process as source of prompt muons: production of charmed states due to cosmic (atmospheric) muons, whose relevant energies range from<sup>2</sup>  $E \sim 100$  GeV up to tenths of TeV’s (for studies in laboratory, see [10]). More specifically, we are concerned with the “open charm” reaction of muoproduction:  $\mu N \rightarrow \mu c\bar{c}X$  ( $X$  denotes a byproduct which does not concern us). This process is stipulated by QED and QCD interactions, while weak interactions provide the instability of charmed states:  $c \rightarrow X_c \rightarrow \mu X$ .

In order to obtain a simple estimate of the flux of double muons due to this process, we adopted the collinear approximation, considering only how the initial muon with energy  $E$  branches into those of the final muons ( $E'$  and  $E''$ ) and proceeded in the following way:

1) First, we calculated the cross section of muoproduction  $d\sigma_{\mu N \rightarrow \mu c\bar{c}X}/dE'dE_c$  at leading order (LO) in  $\alpha_s$ , double differential in the energies of the scattered muon,  $E'$  and of the charm,  $E_c$  (see appendix). This can be done with a limited effort by following the calculations documented in [11], where the cross section integrated over the hadronic phase space  $d\Phi_{hadr}$  was obtained: In fact, neglecting the gluon mass, the differential expression is simply  $d\Phi_{hadr} = dE_c/(8\pi E_\gamma)$ , where  $E_\gamma = E - E'$  is the energy of the virtual photon emitted by the muon<sup>3</sup>. In the actual calculation, that requires integrating over the photon virtuality  $Q^2$  and the gluon momentum fraction  $x$ , we use the GRV98 gluon distribution [12], and set:  $m_c = 1.5, 1.35$  or  $1.2$  GeV. We multiplied the cross section by the factor  $K(E) = \sigma_{NLO}/\sigma_{LO}$  (where  $\sigma$  is the total cross section) to describe next-to leading order QCD effects [13, 14, 15]. The differential cross section increases with  $E'$  with a “ $1/v^2$  behavior” and then rapidly decreases to zero in the range of energies of interest; instead, it is rather mildly distributed in  $E_c$ . The total cross section  $\sigma$  increases with  $E$ , due to the smaller values of  $x$  that are probed by the virtual photon, and to well known characteristics of the gluon distribution. Its value is  $4 \times 10^{-32}$  cm<sup>2</sup> when  $E = 1$  TeV for  $m_c = 1.35$  GeV (almost equal to the trident cross section at the same energy); LO cross section increases by 50% if  $m_c$  is lowered to 1.2 GeV, while decreases by 30% if  $m_c$  is 1.5 GeV.

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<sup>2</sup>We neglect the energy loss in the rock of the charmed hadrons  $X_c$ , for a 200 GeV  $D^\pm$  meson travels on average just 3 cm in the rock before decaying.

<sup>3</sup>Also, we found convenient to relate  $E_c$  to the zenith angle and velocity of emission in the gluon-gamma c.m.s. as follows:  $E_c/E_\gamma = (1 + \beta_c^* \cos \theta_c^*)/2$ , where  $\beta_c^* = \sqrt{1 - 4m_c^2/(p+q)^2}$  ( $p$  and  $q$  are the momenta of the gluon and of the virtual photon)

2) We estimate a “scaling” probability  $dP_{c \rightarrow \mu}/dw \equiv BR_{c \rightarrow \mu} \times \rho_{c \rightarrow \mu}(w)$  that a charm yields a muon with a certain energy fraction  $w = E''/E_c$ , by first hadronizing the charm into a  $D$  meson (using the normalized distribution of [16] with  $\epsilon_D = 0.135$ ) and then letting it decay with a  $K_{\mu 3}$  distribution (that is, retaining only the  $D$  mass, and neglecting the  $Q^2$  dependence of the form factors). The resulting normalized probability  $\rho_{c \rightarrow \mu}(w)$  falls strongly with the energy fraction  $w$ ; the median of the distribution is in fact  $\langle E'' \rangle = 0.15 \times E_c$ . We took as an effective branching ratio of charm into muons the value  $BR_{c \rightarrow \mu} = 8\%$  [17], and multiplied the result by two, to account for the fact that a charm or an anticharm can yield a muon<sup>4</sup>. Notice, incidentally, that the corresponding yield of triple muon is negligible, due to an *a priori* 4% suppression factor.

3) At this stage, we can calculate the cross section  $d\sigma_{\mu N \rightarrow \mu\mu}/dE'dE''$ , where  $E''$  is the energy of the produced muon, and, with that, the cross section<sup>5</sup>  $\sigma_{\mu N \rightarrow \mu\mu}(E, f)$  for production of two muons, each with a fraction of the initial muon energy greater than  $f$ . Due to the behaviors of  $d\sigma_{\mu N \rightarrow \mu c \bar{c} X}$  and  $dP_{c \rightarrow \mu}$  with  $E'$  and  $E''$  mentioned above, this cross section diminishes dramatically with  $f$ ; when  $E = 1$  TeV, it drops down by one order of magnitude already when  $f \approx 0.07$ . This cross section enters the elementary yield of double muons in the detector, which depends linearly on the infinitesimal depth crossed  $dh'$  (in gr/cm<sup>2</sup>):

$$dY_{\mu \rightarrow \mu\mu}(E, h') = dh' \times N_A \times \sigma_{\mu N \rightarrow \mu\mu}(E, f) \quad \text{where } f = \frac{E_{h'}}{E} \quad (1)$$

$N_A = 6.023 \times 10^{23}$  is the number of nucleons in 1 mole (multiplying by  $dh'$ , we obtain the density of targets *per* cm<sup>2</sup>). The energy losses are evaluated in continuous energy loss approximation,  $E_{h'} = (E_h + \epsilon) \exp[(h - h')/h_0] - \epsilon$ , where  $\epsilon \approx 600$  GeV and  $h_0 \approx 2.5$  km w.e. are phenomenological parameters, and  $E_h = 1$  GeV is the (typical) threshold for detection. Multiplying this by the single muon flux differential in  $dE$ ,  $dF_\mu$ , we get the differential double muon flux induced by “open charm” reaction at the depth  $h$ . The total flux is then:

$$F_{\mu\mu}(h) = \int_0^h \int_{2E_{h'}}^\infty dF_\mu(E, h') \times dY_{\mu \rightarrow \mu\mu}(E, h') \quad (2)$$

where we integrated over the depth of production  $h'$ , and the energy  $E$  of the primary muon at this depth (namely, where the reaction takes place);  $E$  was related to the energy at the surface  $E_0$  in the continuous energy loss approximation, which permits us to evaluate  $dF_\mu(E, h')/dE$  by using the (approximate) expression for the flux at the surface given in [18]

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<sup>4</sup>Existing underground detectors do not distinguish between “same charge” and “opposite charge” double muon events.

<sup>5</sup>We consider only those events whose vertex is *not* contained in the detector. Those events profit of a large effective target mass, and correspond, in a sense, to the celebrated neutrino-induced single muon signal.

(namely,  $dF_\mu/dE_0 = 0.14 \times E_0^{-2.7} \times \dots / (\text{cm}^2 \text{ s sr GeV})$ ). The results are shown in the figure, for muons arriving from the vertical direction.

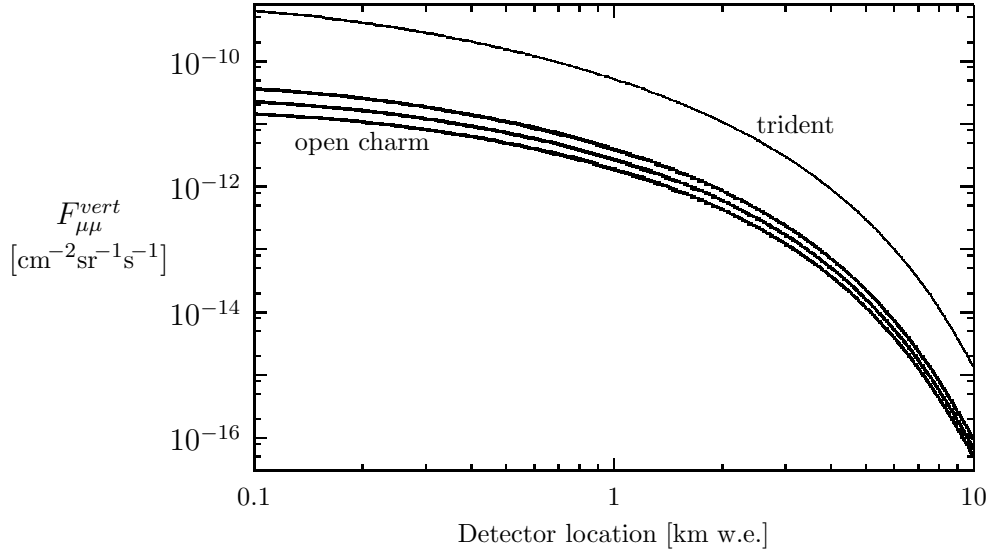


Figure 1: Flux of narrow double muons due to open charm formation (thick curves, for  $m_c = 1.5, 1.35, 1.2$  GeV from lower to higher one) and to the trident reaction (thin curve, calculated with the cross section given in [5]).

The contribution of open charm reaction is not large; for instance, at a depth of 3 km w.e. it is just 4 – 8% of the one due to the trident process. Equivalently, it can be compared with the flux of single muon: we get  $F_{\mu\mu}/F_\mu = 0.7, 1, 1.4 \times 10^{-5}$  in the case of open charm reaction (for  $m_c = 1.5, 1.35, 1.2$  GeV) while  $F_{\mu\mu}/F_\mu = 1.8 \times 10^{-4}$  in the case of trident reaction. For an ideal detector, it would require to accumulate several hundredth (trident narrow double muon) events, to become statistically interesting. The smallness of the result has to be attributed to the relatively small value of  $BR_{c \rightarrow \mu}$ , and to the effective leakage of energy of the virtual photon, during the conversion  $\gamma^* \rightarrow c \rightarrow D \rightarrow \mu$  (while for tridents,  $\gamma^*$  immediately materializes into muons). However, this contribution is not negligible if one aims at reaching the precision of 5 – 10% in the predictions.

The following remarks illustrate other aspects of this result:

- (a) going to shallower depths, the double muon flux increases, though less rapidly than the single muon flux: in fact, the effective target increases with the depth (but, of course, also the background increases);
- (b) conversely, in deeper sites the relative contribution of the open charm process becomes more important, due to more energetic primaries— $E$  increases (but there are practical limitations, due to the time of data taking and area of the installation);
- (c) keeping the depth fixed, and changing the angle of observation, there is an increase of  $F_{\mu\mu}$

moving toward the horizon, directly related to the increase of  $F_\mu$  (but the actual geometry of the rock in the underground site has an essential role in practical considerations);

(d) in water or ice, the trident curve would reduce by  $\sim 1.5$  in comparison with the open charm one, due to the  $Z^2/A$  factor. This would somehow emphasize the open charm contribution (but it should be reminded that no plan exists to have an underwater detector, with large area and capable to achieve a good discrimination at small lateral distances).

In conclusion, it seems to us rather difficult to account for a large fraction of narrow muon bundles on the basis of the open charm process. Thus, the chances of studying heavy quark physics with existing underground (or underwater) detectors are quite limited. This result, however, adds motivations for further search of unexpected sources of background (and, possibly, new sources of prompt muons) when we recall the difficulties to understand existing data on narrow muon bundles. For the future experimental perspectives, we consider interesting the possibility to achieve energy and charge identification of the muons in underground detectors, as a possible handle to separate various components in a narrow muon bundles data set (for instance, we found that the average energy of the parent muon—that forms two muons through the open charm process—is rather large, above 1 TeV). However, even if it will be possible to obtain sufficient statistics and control of the systematics, an attempt to proceed further and extract a signal of production of heavy quarks from studies of narrow muon bundles will need more refined calculations: To accurately describe the NLO effects [14], hadronization, and decay of charmed states [19, 20]; but also those effect in the muon propagation, that are necessary to model the lateral distribution of the events in the underground detectors [21, 22, 6]. In fact, the relatively large transverse momenta  $p_\perp \sim m_c$  that result from charm production and decay lead to larger lateral separations than those due to the trident reaction, and this could be of interest to characterize the charm induced events.

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## Appendix: Formulæ for LO cross sections

The LO cross section, differential in  $y = E_\gamma/E$  and  $z = E_c/E_\gamma$  is:

$$\frac{d\sigma_{\mu N \rightarrow \mu c \bar{c} X}}{dy dz} = \frac{\alpha^2}{9ME} \int d \log Q^2 \int d \log x \alpha_s(\mu^2) G(x, \mu^2) \left[ \left(1 - \frac{2m_\mu^2}{Q^2} + 2 g(y)\right) \frac{df_T}{dz} + \left(1 - \frac{2m_\mu^2}{Q^2} + 6 g(y)\right) \frac{df_L}{dz} \right] \quad (3)$$

where  $M = 0.938$  GeV,  $m_\mu = 0.106$  GeV and  $\alpha = 1/137$ ;  $\alpha_s$  is the strong coupling,  $G$  the gluon distribution function, and  $\mu^2$  the factorization scale ( $\mu^2 = 4m_c^2 + Q^2$  in present calculation). In order to describe nuclear effects in “standard rock” nuclei, we followed [24], and weighted the gluon distribution function by the density:  $r^{s.r.}(x) \approx 1.26 \times x^{0.073} \times (1 - 0.3x)$ ; see eq. 1 and fig. 2 in that work<sup>6</sup>. The adimensional functions introduced in eq. 3 are  $g(y) = (1 - y)/y^2$  and

$$\begin{aligned} \frac{df_T}{dz} &= g_1 - g_0 + x_c g_1 - \frac{x_c^2}{4}(g_2 + 2g_1) - 2x_Q(g_1 - \frac{x_c}{4}g_2) + 2x_Q^2 g_1 \\ \frac{df_L}{dz} &= 2x_Q(g_0 - \frac{x_c}{2}g_1) - 2x_Q^2 g_0 \end{aligned} \quad (4)$$

where  $g_n = 1/z^n + 1/(1 - z)^n$  for  $n = 0, 1, 2$ ;  $x_c = 2m_c^2/pq$  and  $x_Q = Q^2/(2pq)$  are bound by  $x_{min}$  to be lower than unity. The non-trivial limits are: (i)  $x_{min} = (m_c^2 g_1(z) + Q^2)/(2ME_\gamma)$ , which results from the kinematics of the  $\gamma^* g \rightarrow c \bar{c}$  process (setting the gluon mass to zero), and (ii)  $Q_{min}^2 \approx m_\mu^2/g(y)$  which results from setting to zero the scattering angle in the laboratory frame; the limits on  $y$  and  $z$ , and  $Q_{max}^2$  follow by consistence. Notice that the cross section can be easily integrated analytically over  $z$ , which amounts to replace:

$$g_0 \rightarrow 2\beta^*, \quad g_1 \rightarrow 2 \log \frac{1 + \beta^*}{1 - \beta^*}, \quad g_2 \rightarrow \frac{8\beta^*}{1 - (\beta^*)^2} \quad (5)$$

and, also,  $g_1(z) \rightarrow g_1(1/2) = 4$  in the expression of  $x_{min}$ ; the resulting expression for  $d\sigma_{\mu N \rightarrow \mu c \bar{c} X}/dy$  is equivalent to the one shown in the appendix of [11]. The cross section that enters the expression of the double muon yield (eq. 1) is obtained as:

$$\sigma_{\mu N \rightarrow \mu \mu}(E, f) = 2 \times BR_{c \rightarrow \mu} \times \int_f^{1-f} dy \int_{f/y}^1 dz \frac{d\sigma_{\mu N \rightarrow \mu c \bar{c} X}}{dy dz} \times \int_{f/(yz)}^1 dw \rho_{c \rightarrow \mu}(w) \quad (6)$$

The last integral corresponds to an integral probability, and can be tabulated separately to simplify the calculation.

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<sup>6</sup>Notice that here  $x$  is the momentum fraction of the gluon (the particle that feels the nuclear effects), as contrasted with  $x_F \equiv Q^2/(2Pq)$ , the variable introduced to parameterize the structure function  $F^{c\bar{c}}(x_F, Q^2)$ .

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